# Central wavelength and ring pattern of a Fabry-Perot etalon

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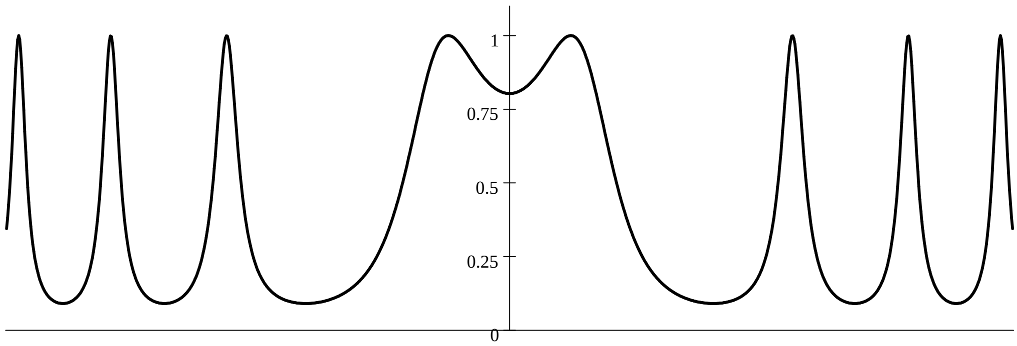
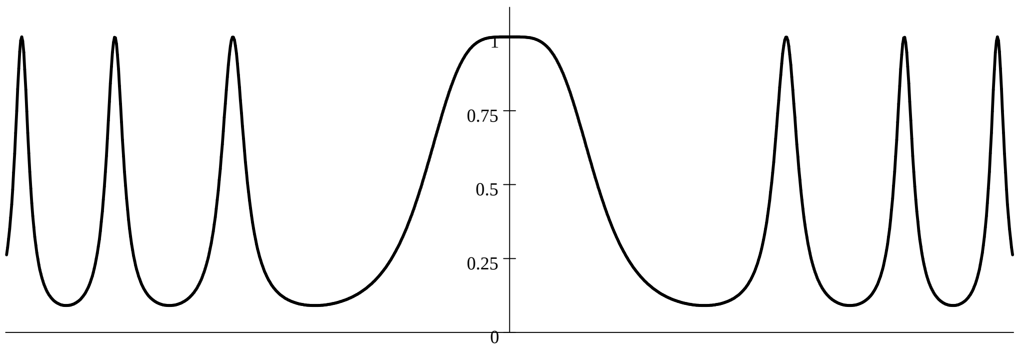
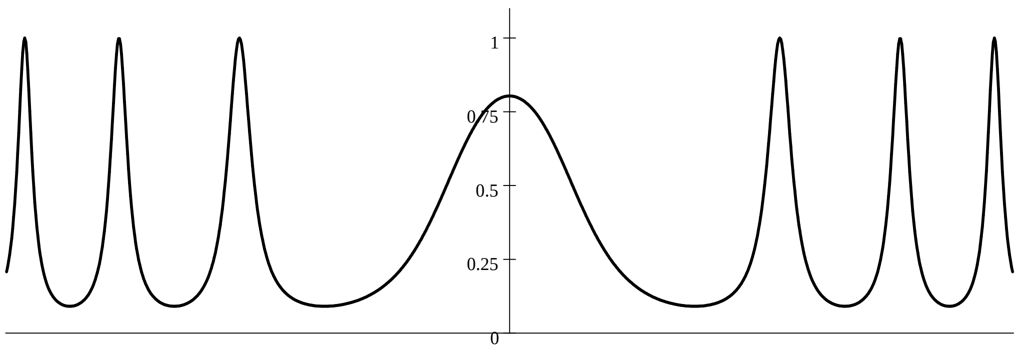
**Case 1 :** $\lambda_0 > \lambda$, $T = \left(1 + 10\sin^2(10.05\pi\cos x)\right)^{-1}$




**Case 2 :** $\lambda_0 = \lambda$, $T = \left(1 + 10\sin^2(10\pi\cos x)\right)^{-1}$




**Case 3 :** $\lambda_0 < \lambda$, $T = \left(1 + 10\sin^2(9.95\pi\cos x)\right)^{-1}$



**The central wavelength of a fabry-Perot etalon**

The **transmission factor** of the etalon is the ratio, between 0 and 100%, of the transmitted intensity divided by its maximum possible value. This factor $T$ is given by the Airy formula, explained in optics textbooks,

$$T = \frac{1}{1 + F\sin^2\left(\frac{2\pi ne\cos r}{\lambda}\right)} \,. \tag{1}$$

The notation is as follows :

- ▶ $F = 4R/(1-R)^2$, where $R$ is the reflection coefficient of the etalon surfaces
- ▶ $n$ is the refractive index of the gap ($n = 1$ for an air-spaced etalon, $n \simeq 1.6$ for a mica-spaced etalon)
- ▶ $e$ is the gap spacing, expressed with the same unit as the light wavelength $\lambda$, thus giving $(2\pi ne\cos r)/\lambda$ in radians.
- ▶ $r$ is the angle of refraction inside the etalon, linked to the angle of incidence $i$ of light on the etalon by Snell's law $\sin r = (\sin i)/n$ (or simply $r \simeq i/n$ if $i$ is small).

A wavelength $\lambda_0$ is called a **central wavelength** (CWL) of the etalon if it is one of the wavelengths of maximum transmission under normal incidence, that is $T = 1$ for $\lambda = \lambda_0$ and $i = r = 0$. This means that $\sin^2(2\pi ne/\lambda_0) = 0$ i.e.

$$2ne = k\lambda_0 \tag{2}$$

for some integer $k$.
When this etalon is lit by a light beam with wavelength $\lambda$ (e.g. a hydrogen lamp with $\lambda = 0.65628$ $\mu$m) its transmission factor is thus

$$T = \frac{1}{1 + F\sin^2\left(k\pi\frac{\lambda_0}{\lambda}\cos r\right)}. \tag{3}$$

For a given etalon, with fixed values of $F$, $n$ and $e$, the accompanying graphs show the variation of $T$ when $i$ (and $r$) vary. Their maxima give the radii of the **ring pattern** produced by the etalon. For the purpose of illustration we have chosen here $F = 10$, $n = 1$ (air-spaced etalon, so that $i = r$), $k = 10$ and the three examples

$$\text{Case 1 : } \lambda_0 = 1.005\lambda \text{ , Case 2 : } \lambda_0 = \lambda \text{ , Case 3 : } \lambda_0 = 0.995\lambda.$$

The first etalon is tuned to a wavelength slightly $> \lambda$, the second to $\lambda$ exactly and the third to a wavelength slightly $< \lambda$.

**Finding the CWL from the ring pattern**

The radii of the ring pattern produced by the etalon correspond to the angles $i$ giving the transmission its maximal value $T = 1$. As the angle $i$ increases from 0, so does the angle $r$ and $k\frac{\lambda_0}{\lambda}\cos r$ decreases, starting from the value $k\frac{\lambda_0}{\lambda}$, close to the integer $k$ if the CWL $\lambda_0$ is close to the lamp wavelength $\lambda$. For the $m$-th ring we have

$$k\frac{\lambda_0}{\lambda}\cos r_m = k - m,$$

where $m \geq 0$ is an integer. If $\lambda_0$ is (slightly) greater than $\lambda$ the first bright ring occurs for $m = 0$, with $\cos r_0 = \lambda/\lambda_0$. This ring is a poorly defined part of the central spot (see the graph for Case 1). If $\lambda_0 \leq \lambda$, the first ring is obtained for $m = 1$ (see Cases 2 and 3).
Thus, leaving aside the case $m = 0$, the ring pattern is defined by

$$\cos r_m = \frac{\lambda}{\lambda_0}\left(1 - \frac{m}{k}\right) \text{ with } m = 1, 2, 3, ... \tag{4}$$

*If the angle of incidence $i_m$ is small enough* we may obtain it by the approximate formula $i_m \simeq d_m/2f$ where $d_m$ is the diameter of the $m$-th bright ring at the prime focus of a camera lens with focal length $f$. Then $r_m \simeq i_m/n$ by Snell's law, $\cos r_m \simeq 1 - (i_m^2/2n^2)$ and (4) becomes

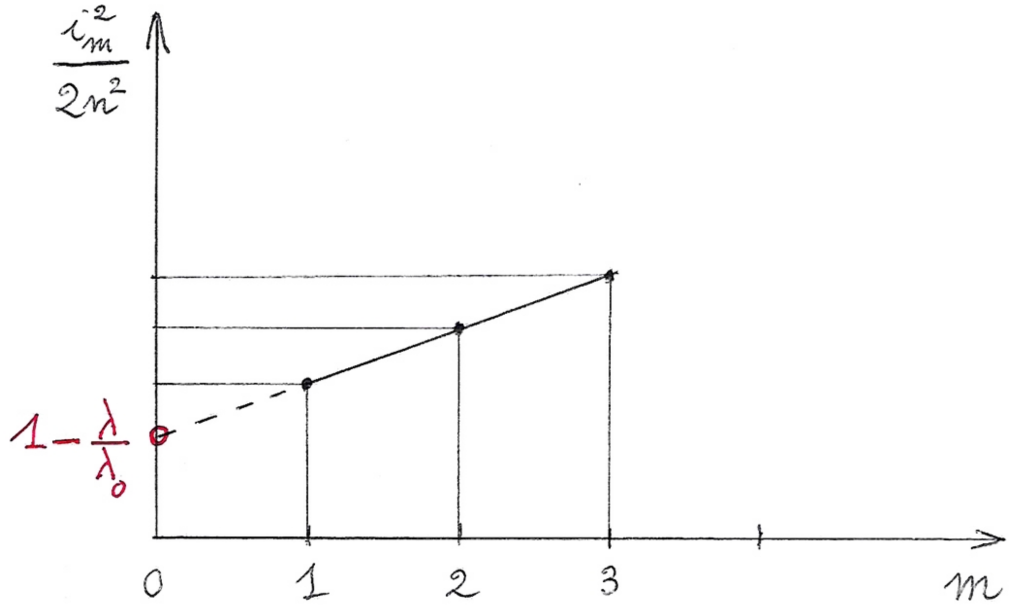
$$\frac{i_m^2}{2n^2} \simeq 1 - \frac{\lambda}{\lambda_0}\left(1 - \frac{m}{k}\right). \tag{5}$$

The graph of $i_m^2/2n^2$ as a function of the ring number $m \geq 1$ is *approximately linear.* When extrapolating it down to $m = 0$, it crosses the vertical axis at $1-(\lambda/\lambda_0)$ (which may be positive or negative) and this gives (approximately) the etalon CWL $\lambda_0$ (see next figure).

**Remark.** If the above approximations aren't good enough, they may be replaced by exact formulas as follows. Considering an air-spaced etalon ($n = 1$) for the sake of simplicity, we have $i_m = r_m$ and (4) becomes

$$\left(1 + \left(\frac{d_m}{2f}\right)^2\right)^{-1/2} = \frac{\lambda}{\lambda_0}\left(1 - \frac{m}{k}\right) \tag{6}$$

(exact formula) where $d_m$ is the diameter of the $m$-th ring measured at the focal plane of the camera lens with focal length $f$. The plot of the left-hand side as a function of $m \geq 1$ is then *exactly linear.* Its extrapolation down to $m = 0$ crosses the vertical axis at $\lambda/\lambda_0$ and this gives the CWL $\lambda_0$. A larger number of rings can now be taken into account for better accuracy.




Approximate measure of the CWL $\lambda_0$ of a Fabry-Perot etalon, by extrapolation from the diameter of its first rings.